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Seismic velocity changes caused by an overburden stress

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ABSTRACT

An increase in seismic velocity with depth is a common rock property, one that can be encountered practically everywhere. Overburden pressure increases vertical stress, producing a nonlinear elastic response. Application of a conventional nonlinear theory to this problem leads to transverse isotropy, with explicit relationships between nonlinear constants and elastic anisotropy parameters. These relationships can be used in velocity "depth trend" removal and in computing offset-dependent corrections for stacking and migration. Assumptions about small static stress and the use of linearized solutions for its evaluation are invalid for overburden problems — more accurate approximations are required. Realistic tomography models should account for elastic anisotropy as a basic feature. Our theory gives an accurate fit to well and stacking velocity data for the Los Angeles Basin. Overburden stress is a likely cause of shear-wave generation by underground explosions.

INTRODUCTION

Increasing seismic velocity with depth is traditionally attributed to a "compaction" of rock caused by overburden pressure (Giles et al., 1998). It is assumed that this compaction is associated not just with porosity changes, which presumably decrease velocity because of the increase in rock density, but rather with some chemical reactions and solid precipitation. However, measurements suggest that velocity consistently increases at all scales starting from tens of meters regionally and extending down to mantle depths, where it exceeds 12 km/s. High velocity gradients at the shallow subsurface act as horizontal waveguides, trapping wave energy and generating increased seismic noise.

Knowledge about velocity distribution is critically important for our ability to image subsurface structures, focus seismic energy at desired points, and retrieve information about underground rock. There is growing evidence that velocity is anisotropic, which has traditionally been explained by the layering that results from geologic sedimentation processes or by a system of fractures with a preferable orientation. It is known that, under high strain, rock elasticity behaves nonlinearly, and this nonlinearity can induce anisotropy (Hughes and Kelly, 1953; Thurston and Brugger, 1964; Thurston, 1965; Nur and Simmons, 1969; Sinha, 1982; Eberhart-Phillips et al., 1989; Norris et al., 1994; Johnson and Rasolofosaon, 1996; Winkler et al., 1998; Fuck and Tsvankin, 2009; Herwanger and Horne, 2009). It is also known that rock stresses can have a tectonic or artificial (laboratory) source. However, the effect of overburden pressure when rock is subjected to a gravitational force caused by an overlying rock mass has not yet been analytically estimated. A single numerical result for this problem is shown in Lei et al. (2012). Here, we apply the equations of five-constant (isotropic) nonlinear elasticity theory to evaluate the effect of overburden pressure on seismic velocity. These equations will first be solved for a distribution of the nonlinear static strain field; then, the result will be used to estimate its effect on the propagation of seismic waves. It will be shown that the overlying rock causes significant transverse isotropy, which explains the observed increase in P- velocity with depth. The effect of shear-wave generation by an explosive source in such a medium will also be evaluated.

DYNAMIC AND STATIC FIELD COMPONENTS

We consider a nonlinear elastic medium with Lamé parameters λ, μ, and density ρ originally described by Murnaghan (1951). His theory requires the introduction of three third-order elastic (TOE) constants A, B, and C, using the notation of Landau and Lifshitz (1953). In an unstressed state, the medium is isotropic. We are interested in the effective properties of the medium when it is subjected to a static (time-invariable) stress.

Assuming that the elastic deformation in a solid with displacement vector,

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$$\mathbf{u} = \mathbf{u}(x, y, z) = \mathbf{u}(x_1, x_2, x_3), \tag{1}$$

is continuous, together with its spatial derivatives, the stress components have the form (Zarembo and Krasilnikov, 1966),

$$\begin{aligned}\sigma_{ik} = {} & \lambda \frac{\partial u_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \\ & + \left(\mu + \frac{A}{4}\right)\left(\frac{\partial u_s}{\partial x_i}\frac{\partial u_s}{\partial x_k} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_s}{\partial x_k}\frac{\partial u_i}{\partial x_s}\right) \\ & + \frac{(B+\lambda)}{2}\left(\left(\frac{\partial u_s}{\partial x_j}\right)^2\delta_{ik} + 2\frac{\partial u_i}{\partial x_k}\frac{\partial u_s}{\partial x_s}\right) + \frac{A}{4}\frac{\partial u_k}{\partial x_s}\frac{\partial u_s}{\partial x_i} \\ & + \frac{B}{2}\left(\frac{\partial u_s}{\partial x_j}\frac{\partial u_j}{\partial x_s}\delta_{ik} + 2\frac{\partial u_k}{\partial x_i}\frac{\partial u_s}{\partial x_s}\right) + C\left(\frac{\partial u_s}{\partial x_s}\right)^2\delta_{ik}. \end{aligned} \tag{2}$$

(Here and below, repeated indexes s and j signify summation.)

Taking into account that, for known elastic materials, the Lamé constants are by several orders of magnitude smaller than the absolute values of the constants A, B, and C, the stress components from equation 2 can be reduced to

$$\begin{aligned}\sigma_{ik} = {} & \lambda \frac{\partial u_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \\ & + \frac{A}{4}\left(\frac{\partial u_s}{\partial x_i}\frac{\partial u_s}{\partial x_k} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_s}{\partial x_k}\frac{\partial u_i}{\partial x_s} + \frac{\partial u_k}{\partial x_s}\frac{\partial u_s}{\partial x_i}\right) \\ & + \frac{B}{2}\left(\left(\frac{\partial u_s}{\partial x_j}\right)^2\delta_{ik} + 2\frac{\partial u_i}{\partial x_k}\frac{\partial u_s}{\partial x_s} + \frac{\partial u_s}{\partial x_j}\frac{\partial u_j}{\partial x_s}\delta_{ik} + 2\frac{\partial u_k}{\partial x_i}\frac{\partial u_s}{\partial x_s}\right) \\ & + C\left(\frac{\partial u_s}{\partial x_s}\right)^2\delta_{ik}. \end{aligned} \tag{3}$$

The equations of motions then have the form,

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k} + F_i, \tag{4}$$

where t is time and F_i is an external force acting per unit volume.

We assume that the total field $\mathbf{u}$ consists of the static component $\mathbf{U}$ arising from an applied static stress and a time-dependent component $\mathbf{w} = \mathbf{w}(t)$, which describes propagating waves

$$\mathbf{u} \approx \mathbf{U} + \mathbf{w}, \tag{5}$$

where it is also assumed that the amplitudes of the related static strains significantly exceed those related to the dynamic field $\mathbf{w}$. These assumptions allow us to neglect square terms containing components of $\mathbf{w}$ after substitution of equation 5 into equation 3. Consideration of such terms leads to multiple harmonics generation (Gol'dberg, 1961; Polyakova, 1964; Korneev et al., 1998). Similar to the displacement field, the stress components can also be separated into static and dynamic parts:

$$\sigma_{ik} \approx \bar{\sigma}_{ik} + \tilde{\sigma}_{ik}. \tag{6}$$

Retention of just static stress components in equation 3 gives the equation,

$$\begin{aligned}\bar{\sigma}_{ik} = {} & \lambda \frac{\partial U_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial U_i}{\partial x_k} + \frac{\partial U_k}{\partial x_i}\right) \\ & + \frac{A}{4}\left(\frac{\partial U_s}{\partial x_i}\frac{\partial U_s}{\partial x_k} + \frac{\partial U_k}{\partial x_s}\frac{\partial U_i}{\partial x_s} + \frac{\partial U_s}{\partial x_k}\frac{\partial U_i}{\partial x_s} + \frac{\partial U_k}{\partial x_s}\frac{\partial U_s}{\partial x_i}\right) \\ & + \frac{B}{2}\left(\left(\frac{\partial U_s}{\partial x_j}\right)^2\delta_{ik} + 2\frac{\partial U_i}{\partial x_k}\frac{\partial U_s}{\partial x_s} + \frac{\partial U_s}{\partial x_j}\frac{\partial U_j}{\partial x_s}\delta_{ik} + 2\frac{\partial U_k}{\partial x_i}\frac{\partial U_s}{\partial x_s}\right) \\ & + C\left(\frac{\partial U_s}{\partial x_s}\right)^2\delta_{ik}, \end{aligned} \tag{7}$$

where $\bar{\sigma}_{ik}$ obeys the equation

$$\frac{\partial \bar{\sigma}_{ik}}{\partial x_k} + F_i = 0, \quad i = 1, 2, 3. \tag{8}$$

After substituting equation 5 into equation 3 and keeping terms linear with respect to $\mathbf{w}$, we obtain

$$\begin{aligned}\tilde{\sigma}_{ik} = {} & \lambda \frac{\partial w_s}{\partial x_s}\delta_{ik} + \mu\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right) \\ & + \frac{A}{4}\left(\frac{\partial U_s}{\partial x_i}\frac{\partial w_s}{\partial x_k} + \frac{\partial w_s}{\partial x_i}\frac{\partial U_s}{\partial x_k} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_i}{\partial x_s}\right. \\ & \left. + \frac{\partial U_s}{\partial x_k}\frac{\partial w_i}{\partial x_s} + \frac{\partial w_s}{\partial x_k}\frac{\partial U_i}{\partial x_s} + \frac{\partial U_k}{\partial x_s}\frac{\partial w_s}{\partial x_i} + \frac{\partial w_k}{\partial x_s}\frac{\partial U_s}{\partial x_i}\right) \\ & + B\left(\frac{\partial U_s}{\partial x_j}\frac{\partial w_s}{\partial x_j}\delta_{ik} + \frac{\partial U_i}{\partial x_k}\frac{\partial w_s}{\partial x_s} + \frac{\partial w_i}{\partial x_k}\frac{\partial U_s}{\partial x_s} + \frac{\partial U_s}{\partial x_j}\frac{\partial w_j}{\partial x_s}\delta_{ik}\right. \\ & \left. + \frac{\partial U_k}{\partial x_i}\frac{\partial w_s}{\partial x_s} + \frac{\partial w_k}{\partial x_i}\frac{\partial U_s}{\partial x_s}\right) + 2C\frac{\partial U_j}{\partial x_j}\frac{\partial w_s}{\partial x_s}\delta_{ik}. \end{aligned} \tag{9}$$

If we solve equation 8 and determine "static strains" $\partial U_i/\partial x_k$, $(i, k = 1, 2, 3)$, they can then be substituted into equation 9 to arrive at an equation of motion that is linear with respect to the dynamic field $\mathbf{w}$,

$$\rho\frac{\partial^2 w_i}{\partial t^2} = \frac{\partial \tilde{\sigma}_{ik}}{\partial x_k}, \tag{10}$$

which describes a linear anisotropic medium.

UNIAXIAL STRESS

Here, we analyze a uniaxial stress case that leads to a transverse isotropic (TI) medium. We consider a uniaxial force acting normally on a plane $z = \text{const}$,

$$\mathbf{F} = (0, 0, F_3), \tag{11}$$

along the OZ axis. Assuming that the fields do not depend on lateral dimensions x and y (unbounded case), from equations 7, 8, and 11, we obtain

$$(\lambda + 2\mu)\frac{\partial U_3}{\partial x_3} + 2(A + 3B + C)\left(\frac{\partial U_3}{\partial x_3}\right)^2 = -F_3, \tag{12}$$

which can be solved exactly as a square polynomial, and

$$q \equiv \frac{\partial U_3}{\partial x_3} = -\frac{\lambda + 2\mu}{4(A+3B+C)} \times \left(1 - \sqrt{1 - \frac{8(A+3B+C)F_3}{(\lambda+2\mu)^2}}\right), \quad (13)$$

which defines the only nonzero static-strain component. The choice of the $(-)$ sign before the radical in equation 13 provides obvious linear solution with respect to the applied force,

$$q \approx q_0 = -\frac{F_3}{\lambda + 2\mu}, \quad \text{when} \quad \left|\frac{8(A+3B+C)F_3}{(\lambda+2\mu)^2}\right| \ll 1, \quad (14)$$

for the case when the nonlinear static deformation is negligible. The assumption of a linear static strain has been used in all previous publications on this subject, but as it is demonstrated in the numerical example below, this assumption is not justified for stresses caused by an overburden.

For high deformations (large depths) and/or large TOE constants, the equation has an asymptotic solution,

$$q \asymp q_\infty = \sqrt{\frac{-F_3}{2(A+3B+C)}}, \quad \text{when} \quad \left|\frac{8(A+3B+C)F_3}{(\lambda+2\mu)^2}\right| \gg 1. \quad (15)$$

Note that quantity $A + 3B + C$ should always be negative.

Substituting solutions 13 into equation 8, we obtain the dynamic stress components,

$$\tilde{\sigma}_{11} = \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{11} + 2q((B+C)\varepsilon_{11} + C\varepsilon_{22} + (B+C)\varepsilon_{33}), \quad (16)$$

$$\tilde{\sigma}_{22} = \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{22} + 2q(C\varepsilon_{11} + (B+C)\varepsilon_{22} + (B+C)\varepsilon_{33}), \quad (17)$$

$$\tilde{\sigma}_{33} = \lambda(\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}) + 2\mu\varepsilon_{33} + 2q((B+C)\varepsilon_{11} + (B+C)\varepsilon_{22} + (A+3B+C)\varepsilon_{33}), \quad (18)$$

$$\tilde{\sigma}_{13} = 2\left(\mu + q\left(\frac{A}{2} + B\right)\right)\varepsilon_{13}, \quad (19)$$

$$\tilde{\sigma}_{23} = 2\left(\mu + q\left(\frac{A}{2} + B\right)\right)\varepsilon_{23}, \quad (20)$$

$$\tilde{\sigma}_{12} = 2(\mu + qB)\varepsilon_{12}, \quad (21)$$

where

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial w_i}{\partial x_k} + \frac{\partial w_k}{\partial x_i}\right). \quad (22)$$

For a TI elastic media, the stress and strain components have the relationship,

$$\begin{pmatrix} \tilde{\sigma}_{11} \\ \tilde{\sigma}_{22} \\ \tilde{\sigma}_{33} \\ \tilde{\sigma}_{23} \\ \tilde{\sigma}_{13} \\ \tilde{\sigma}_{12} \end{pmatrix} = C_{\alpha\beta} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{pmatrix}, \quad (23)$$

through the symmetric matrix,

$$C_{\alpha\beta} = \begin{pmatrix} \lambda' + 2\mu' & \lambda' & \lambda' - l & 0 & 0 & 0 \\ \lambda' & \lambda' + 2\mu' & \lambda' - l & 0 & 0 & 0 \\ \lambda' - l & \lambda' - l & \lambda' + 2\mu' - p & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu' - m & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu' - m & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu' \end{pmatrix}, \quad (24)$$

where λ' and μ' are the effective Lamé parameters of the TI medium and l, p, and m are anisotropy coefficients. Comparing equation 23 with equations 16–21, we derive the following relationships between the elastic parameters of TI and the original nonlinear media:

$$\lambda' = \lambda + 2qC, \quad (25)$$

$$\mu' = \mu + qB, \quad (26)$$

$$l = -2qB, \quad (27)$$

$$m = -q\frac{A}{2}, \quad (28)$$

$$p = -2q(A + 2B). \quad (29)$$

Thus, the uniaxially stressed nonlinear medium is effectively a TI medium with parameters given by equations 25–29.

Elastic waves in TI media are a well-studied matter. One P- and two S-waves can propagate when anisotropy is present, and their velocities are generally complex functions of elastic moduli. According to Petrashen (1980), in TI media for the particular values 0 and $\pi/2$ of the angle θ between the OZ axis and the direction of propagation, the velocities of these three waves have simple forms:

$$V_P^2(0) = \frac{\lambda' + 2\mu' - p}{\rho}, \quad (30)$$

$$V_P^2\left(\frac{\pi}{2}\right) = \frac{\lambda' + 2\mu'}{\rho}, \quad (31)$$

$$V_{S1}^2(0) = V_{S1}^2\left(\frac{\pi}{2}\right) = \frac{\mu' - m}{\rho}, \quad (32)$$

$$V_{S2}^2(0) = \frac{\mu' - m}{\rho}, \quad (33)$$

$$V_{S2}^2\left(\frac{\pi}{2}\right) = \frac{\mu'}{\rho}. \quad (34)$$

All propagation velocities from equations 30–34 depend on the static stress through the elastic constants, all of which are functions of parameter q.

Using equation 23, we also can account for the density changes caused by finite deformation. This can be done using the approximate formula

$$\rho = \frac{\rho_0}{1 + 2q}, \quad (35)$$

where ρ_0 is the density in an unstressed state (Hughes and Kelly, 1953). Equation 35 can be solved together with equation 12, giving a polynomial of the third order. However, numerical evaluations show that the corresponding changes in velocity do not exceed more than a few percent, so that such corrections are unnecessary.

OVERBURDEN STRESS

Assume that the uniaxial stress has a gravitational origin. Then, for constant density material,

$$F = \rho g z, \quad (36)$$

where g is the gravitational acceleration constant and z is a depth. The assumption about constant density is not critically important for the formulation; if the density varies with depth, it should be integrated along depth in equation 36. From equations 30–34 and 25–29, we have

$$V_P^2(0) = V_{P0}^2 + \frac{2q(A + 3B + C)}{\rho}, \quad (37)$$

$$V_P^2\left(\frac{\pi}{2}\right) = V_{P0}^2 + \frac{2q(B + C)}{\rho}, \quad (38)$$

$$V_{S1}^2(0) = V_{S1}^2\left(\frac{\pi}{2}\right) = V_{S2}^2(0) = V_{S0}^2 + \frac{q(B + A/2)}{\rho}, \quad (39)$$

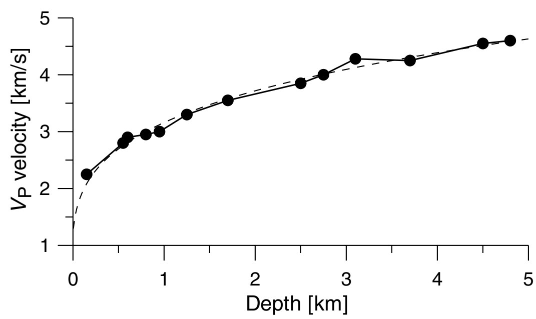

Figure 1. Average velocity model for the Los Angeles Basin (solid line with marked data points) and the least-squares fit (dashed line) using equation 37.

$$V_{S2}^2\left(\frac{\pi}{2}\right) = V_{S0}^2 + \frac{qB}{\rho}. \quad (40)$$

Remarkably, the nonlinear coefficient $A + 3B + C$ of a vertically propagating P-wave is the same as for a nonlinear component (multiple harmonic) of this wave (Gol'dberg, 1957). Equations 37–40 suggest that for small deformation, when solution 14 can be used, the squares of wave velocities are linear functions of depth.

At large depths (and/or strong nonlinearity), when approximation 15 is valid, the velocities V from equations 37–40 have depth-dependence in the form,

$$V \sim z^{1/4}. \quad (41)$$

SHEAR WAVE GENERATION BY A POINT-PRESSURE SOURCE

The amplitudes of excited elastic waves also depend on static stress. For the case of a point-pressure source in a TI medium, P- and S-waves can be generated. Using the result of Kiselev (2001), we can put the ratio between amplitudes of P- and S-waves for weak anisotropy in the form

$$\frac{|\mathbf{u_S}|}{|\mathbf{u_P}|} = \left(\frac{\lambda' + 2\mu'}{\mu'}\right)^{3/2} \frac{|q(A + 2B)|}{2(\lambda' + \mu')} \sin 2\theta$$
$$\approx \left(\frac{\lambda + 2\mu}{\mu}\right)^{3/2} \frac{|q(A + 2B)|}{2(\lambda + \mu)} \sin 2\theta, \quad (42)$$

stating that the amplitude ratio of S- to P-waves is proportional to the static stress in the medium. However, the results of the next section show that anisotropy caused by overburden does not let to consider it as weak.

NUMERICAL EXAMPLES

To evaluate the derived analytical formulas, we use the results from Suss and Shaw (2003), who analyzed more than 150 sonic logs and 7000 stacking velocities from industry reflection profiles at regional scale for the Los Angeles Basin. A comparison of the average velocity for the areas with variance errors below 0.8 and a least-squares fit of equation 37 is shown in Figure 1. Nonlinear static strain q is computed using the exact equation 13. Assuming $V_S/V_P = 0.55$, the following values for the elastic constants are obtained: $\lambda = 1.9$ GPa, $\mu = 0.97$ GPa, $A + 3B + C = -10.8 \cdot 10^3$ GPa. Using the ratios of six independent results for the measured nonlinear constants in Berea sandstone presented in Table 3 from Sarkar et al. (2003), we evaluate the nonlinear elastic constants as $A = -1.1 \cdot 10^3$ GPa, $B = -2.9 \cdot 10^3$ GPa, $C = -1 \cdot 10^3$ GPa. These values are used in the next numerical examples.

Figure 2 shows the ratio q_0/q of the linear static strain from equation 14 to the exact solution 13 of equation 12. Figure 3 shows the depth-dependence of velocities from equations 37–40. The predicted ratio between vertical and horizontal velocities as a function of depth is shown in Figure 4.

DISCUSSION

Anisotropy in rock can occur not just because of an applied stress, but also due to structuring on microscopic (clays) and mesoscopic (sedimentary layering) levels. The combined contribution of all causes could either reduce or increase the overall effect. However, only overburden stress is a common phenomenon, which causes TI anisotropy and exists practically everywhere at all depths. Evaluation shows (Figures 3 and 4) that such anisotropy can be significant, reaching several tens of percent and probably more. This result is consistent with common practices, when stacking velocity requires extra corrections with a changing offset. This also implies that, at field scales, all rock is likely to be anisotropic, and this anisotropy needs to be accounted for during migration of data, tomography, amplitude-versus-offset analysis, etc. (Bui et al., 2011).

Nonlinear elastic coefficients can have different relationships with rock parameters (Brekhovskikh, 1980; Sayers and Kachanov, 1995; Sinha and Kostek, 1996; Prasad and Manghnani, 1997; Sinha and Plona, 2001; Prioul et al., 2004; Shapiro and Kaselow, 2005; Sayers, 2006; Koesoemadinata et al., 2010; Gurevich et al., 2011), depending on rock fracturing, microstructure, and saturation. Inversion of anisotropy in stressed rock (Sarkar et al., 2003) can evaluate

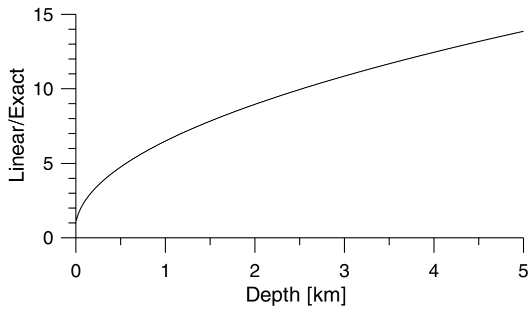

Figure 2. Ratio between linearized (equation 14) and exact (equation 13) solutions for the static strain for uniaxial load.

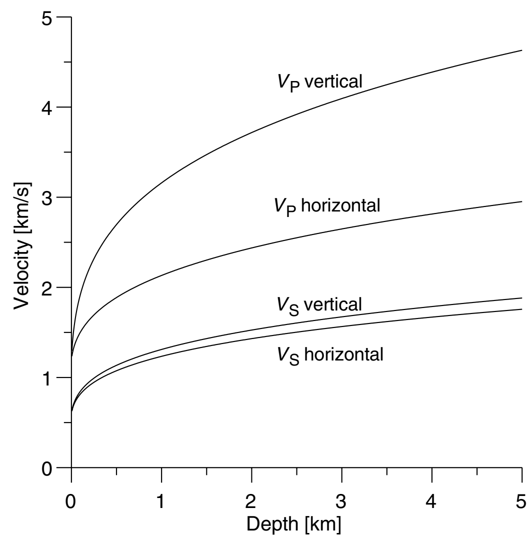

Figure 3. Estimated P- and S-wave velocities induced by overburden pressure.

the nonlinear coefficients, which can be interpreted through assumptions regarding the concrete mechanism of rock nonlinearity. The results of this study can be used for such problems if the nonlinear rock model obeys equations 2, 3, and 4, i.e., if the deformation is recoverable after a stress release, no hysteretic behavior is assumed.

In all previous publications on the subject, the assumption of a small static strain was made to determine the elastic effect caused by stress. Figure 2 demonstrates that, for overburden stress effects, this assumption is not justified, and a more accurate solution is needed. For uniaxial stress, this solution is explicit and given by equation 13. For more complex external forces, equation 7 can be solved numerically to find all static stress components, which then can be used in equation 23. An accurate estimate of static strain is important, not just for the proper evaluation of velocity changes, but also for making corrections related to propagation distance change (Fuck et al., 2011).

Thus, the same rock at different depths might have different wave-propagation properties. Proper geologic interpretation of seismic velocity maps requires application of local depth corrections that can remove the overburden-pressure effects. Laboratory velocity measurements must be corrected as well for nonlinear overburden effects when applied to field scales. Nonlinear rock coefficients can be determined from special laboratory measurements. They can also be evaluated by observing nonlinear propagation effects, such as multiple frequency generation.

The results obtained in this paper for seismic velocities as functions of uniaxial stress differ from those of Hughes and Kelly (1953), even for small nonlinearity when the solution of equation 13 can be expressed in the linearized form 14. Derivation in Hughes and Kelly (1953) is very sketchy and lacks some important details. At the same time, they find that "it is most difficult in this subject to keep a strict level of accuracy, and several apparent violations occur in the literature." Given the lengthy derivations, it is important to independently verify the final result, if possible.

We believe that the results obtained here are valid, for the following reasons. First, the initial equation 2 for stress and the related equations of motions for an isotropic nonlinear elasticity can be

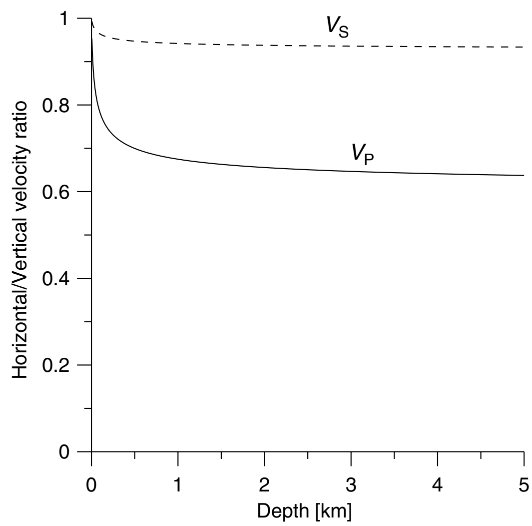

Figure 4. The ratio between vertical and horizontal velocities as a function of depth for P- (lower curve) and S- (upper curve) waves.

found in several standard publications on the subject (Landau and Lifshitz, 1953; Gol'dberg, 1961; Jones and Kobett, 1963; Taylor and Rollings, 1964; Zarembo and Krasilnikov, 1966), where they have been rederived and verified. Second, the effects of stress for the general (anisotropic) case of elastic nonlinearity are studied by Fuck and Tsvankin (2009). They used TOE tensors, expressed in Voigt notations represented by $6 \times 6 \times 6$ matrices. Equations 23–32 in this paper are verified by comparison with their equation 46 and using the following relationships:

$$\lambda = C_{12}, \mu = C_{66},\ A = \frac{1}{2}(C_{111} - 3C_{112} + 2C_{123}),$$
$$B = \frac{1}{2}(C_{112} - C_{123}),\ C = \frac{1}{2}C_{123}. \qquad (43)$$

Third, the validity of the equations for velocities in a TI medium, by Petrashen (1980), has been verified by independent authors (Musgrave, 1970; Helbig, 1994; Tsvankin, 2005).

The presented results indicate that at large depths, depth dependence follows the law $V \sim z^{1/4}$ (equation 41). After analyzing velocity data from 500 well surveys in the United States and Canada, Faust (1951) suggests an empirical formula $V = 125.3(ZT)^{1/6}$, for average shales and sands, where V is velocity in feet per second, Z is depth in feet, and T is age in years. Of course, the age factor positively correlates with depth. On the other hand, we can obtain $V \sim T^{1/6}$ dependence also by adding cubic terms into the stress-strain equation, and the application of the fourth-order elastic constants is needed. The relationships (equations 25–29) between the nonlinear elastic constants and the elastic constants of the effective TI medium enable estimates of the medium anisotropy induced by an applied stress, if all velocities are being measured.

Evaluations of S-wave emissions from point-pressure sources due to overburden pressure cannot make the assumption of weak anisotropy. Significant S-wave excitation has comparable amplitudes to the P-wave — a well-known problem in explosion seismology (Leavy, 1993; Imhof and Toksöz, 2000; Liu and Ahrens, 2001). However, formula 42 (above) was derived from an assumption of weak anisotropy, and the obtained ratio is likely to deviate from an exact solution. This problem should be addressed using numerical methods such as those of Preston and Aldridge (2011). Solving this problem might enable remote stress measurements using underground explosions.

Finally, the widely used Thomsen (1986) parameters for a weak TI medium have the following expressions using the coefficients l, p, and m:

$$\varepsilon = \frac{p}{\lambda + 2\mu - p}, \quad \gamma = \frac{m}{\mu - m},$$
$$\delta = \frac{(l + p - 2m)(2\lambda + 2\mu - l - p)}{(\lambda + 2\mu - p)(\lambda + \mu - p + m)}. \qquad (44)$$

CONCLUSIONS

The increase in seismic velocity with depth is a common property of rock, one that can be encountered practically everywhere. Overburden pressure increases vertical stress, producing nonlinear elastic responses. Application of nonlinear theory to this problem leads to transverse isotropy, with relatively simple relationships between the nonlinear constants and anisotropy elastic coefficients. These relationships can be used in velocity depth trend removal and in computing offset-dependent corrections for stacking and migration. This also implies that realistic tomography models should account for elastic anisotropy as a basic feature. A proper solution for overburden stress requires a full nonlinear solution for static stress distribution.

It is quite likely that anisotropy resulting from overburden pressure is a common basic property of underground rock. Accounting for anisotropy properties requires more complex computational and imaging tools than just isotropic models, which have generally been in use up to the present. On the other hand, seismic interpretation can arrive at additional imaging rock parameters (TOE constants), which can potentially be extracted from anisotropy measurements. Additionally, the overburden-induced anisotropy is strong enough to produce shear waves by an explosion.

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